Engineering Notes

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The Aerodynamic Behavior of Infinite Swept Wings: Another Point of View

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Introduction

THE problem of the aerodynamic behavior of infinite swept wings in incompressible flow is well known and appears in textbooks dealing with the aerodynamics of wings. Using thin-airfoil theory and comparing cross sections that are parallel to the freestream direction with cross sections that are perpendicular to the spanwise direction, the following conclusion is reached: the lift curve slope of cross sections, parallel to the freestream direction of infinite swept wings, is reduced by a factor that is equal to the cosine of the sweep angle. This result appears in different excellent text books, of which Refs. 1-3 are outstanding examples.

When these results are examined, one is frequently faced with the problem that although the derivation is clear and correct, there is not any convincing physical explanation to this reduction in the lift curve slope that is an inherent property of the two dimensional profile. Therefore, the fact must be accepted although there is no satisfactory explanation.

The purpose of the present note is to introduce another point of view of the behavior of infnite swept wings. According to the present explanation, the reduction of the lift per unit length of the wing is not due to a reduction of the lift curve slope of the two dimensional profiles, but is due to reduction in the effective angle of attack of each cross section. This reduction in the effective angle of attack is a result of changes in the vorticity field of the swept wing (compared to the unswept wing) that result in changes in the velocities induced over the wing.

Theoretical Derivation

In Fig. 1, an infinite swept wing is shown. Three different systems of coordinates are defined. The first system is the x-y system, where U, the freestream velocity, is in the x direction. Using the classical vortex theory, the swept wing is represented by a vortex sheet covering the wing projection onto the x-y plane. The second system of coordinates is the ξ-η system. This is a local nonorthogonal system of coordinates where ξ is parallel to the x coordinate, while η points in the spanwise direction. If Λ is the sweep angle, then (see Fig. 1)

$$\xi = x - y\tan\Lambda \quad \eta = y/\cos\Lambda \tag{1}$$

The third system of coordinates is the ζ-μ system. The direction of μ is identical to that of η, while ζ is orthogonal to μ.

In the present problem, one can think of two kinds of cross sections. It is possible to take cross sections that are parallel to U. These cross sections have a chord c (see Fig. 1) and angle of pitch α. The second kind of cross sections are those that are perpendicular to the spanwise direction of the wing (they are in the ζ direction). The chord and angle of pitch of the second kind of cross sections are $\bar{c}$ and $\bar{\alpha}$, respectively. It is clear from simple geometric reasoning that (for the case of small α):

$$\bar{c} = c\cos\Lambda \quad \bar{\alpha} = \alpha/\cos\Lambda \tag{2}$$

The vortex sheet is described by its two components, γ_a and δ_a. γ_a is the circulation per unit distance in the x direction taken about the positive y direction. δ_a is the circulation per unit distance in the y direction, taken about the negative x direction. At any point of the wing,

$$\frac{\partial \gamma_a}{\partial y} = \frac{\partial \delta_a}{\partial x} \tag{3}$$

Since the wing is infinite, it is clear that, at all the cross sections parallel to U, the distributions of γ_a and δ_a are the same. This means that γ_a and δ_a are functions of ξ, but not functions of η. Using Eqs. (1) and (3):

$$\frac{\partial \delta_a}{\partial x} = \frac{\partial \delta_a}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \delta_a}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial \delta_a}{\partial \xi} = \frac{\partial \gamma_a}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial \gamma_a}{\partial \eta}\frac{\partial \eta}{\partial y} = -\frac{\partial_{\gamma_a}}{\partial \xi} tg\Lambda \tag{4}$$

Integration of Eq. (4) with respect to ξ implies

$$\delta_a = -\gamma_a \tan\Lambda \tag{5}$$

Equation (5) indicates that γ_a and δ_a can be replaced by a resultant vorticity ϵ that is directed in the positive η direction. If ϵ is measured per unit length in the ζ direction, then

$$\epsilon = \gamma_a/\cos\Lambda \tag{6}$$

The velocity that is induced by the vortex sheet at each point of the wing, perpendicular to the x, y plane, is denoted w. It is clear that since the wing is uniform and infinite, the chordwise distribution of w (along c or $\bar{c}$) is the same for any cross section of the wing. The pattern of induced velocities must meet the linearized boundary condition

$$U\alpha = w \tag{7}$$

The chordwise distribution of γ_a and δ_a must induce a velocity that satisfies Eq. (7). In addition, they also must satisfy the Kutta hypothesis at the trailing edge. Equation (7) can also be written in a different form:

$$(U\cos\Lambda)(\alpha/\cos\Lambda) = (U\cos\Lambda)\bar{\alpha} = w \tag{8}$$

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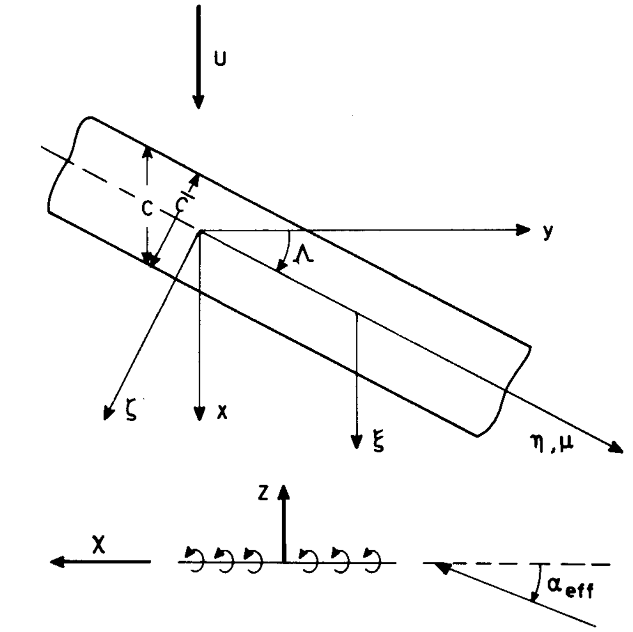

Fig. 1 General description of the problem.

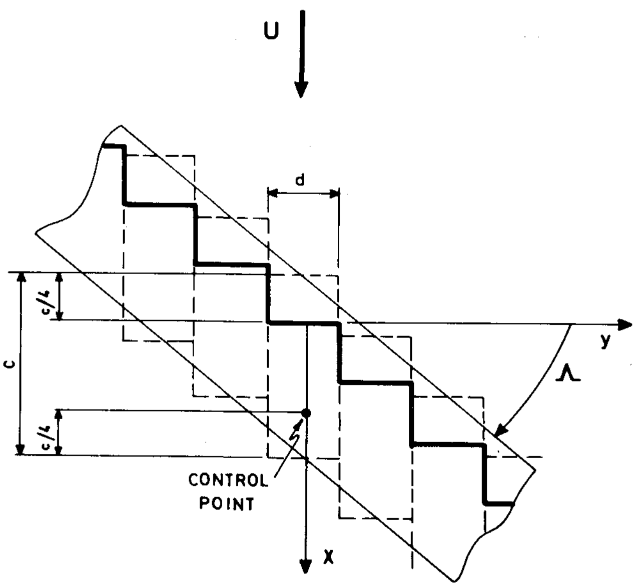

Fig. 2 Discretization scheme.

The vorticity distribution ϵ which satisfied Eq. (8) (and also satisfied Kutta's hypothesis) is well known and is the chordwise vorticity distribution of a two-dimensional profile positioned at a pitch angle $\bar{\alpha}$, while the freestream velocity is $U\cos\Lambda$. We have obtained here the equivalence between an infinite wing that is swept at an angle Λ (where the cross sections parallel to the freestream velocity have a pitch angle α and chord c) and an unswept wing (where the freestream velocity is $U\cos\Lambda$, the pitch angle $\alpha/\cos\Lambda$, and the chord $c\cos\Lambda$).

For any cross section of the wing, the chordwise vorticity distribution v per unit length is given by the so-called flat plate chordwise loading:

$$v=2V\alpha_{\text{eff}}\sqrt{(1-\chi)/(1+\chi)} \tag{9}$$

As shown in Fig. 1, U is the planar freestream velocity, α_{eff} is the effective angle of attack, and χ is a nondimensional chordwise coordinate that originates at the midchord and is equal to -1 and 1 at the leading and trailing edges, respectively. If Eq. (9) is applied to a cross section perpendicular to η, then

$$V\equiv U\cos\Lambda \qquad \alpha_{\text{eff}}\equiv\bar{\alpha}\equiv\alpha/\cos\Lambda \qquad v\equiv\epsilon \tag{10}$$

For a cross section parallel to U

$$V\equiv U \qquad v\equiv\gamma_a \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (9) and using Eq. (6) imply that $\alpha_{\text{eff}x}$ (the effective angle of attack of cross sections that are parallel to U) is given by

$$\alpha_{\text{eff}x}=\alpha\cos\Lambda \tag{12}$$

Equation (12) indicates that in the case of swept wings, if cross sections parallel to the freestream velocity are considered, the effective angle of attack is not the pitch angle α, but is given by Eq. (12). *It is not the sectional lift curve slope that is changed owing to the sweep, but the effective angle that is reduced by a factor of cos*Λ *compared with the geometric pitch.* This change in the effective angle of attack is a result of the changes in the vortex sheet compared with the case of infinite unswept wing. These changes include a relative translation of the cross sections and the presence of the chordwise vorticity component δ_a. This concept of in-

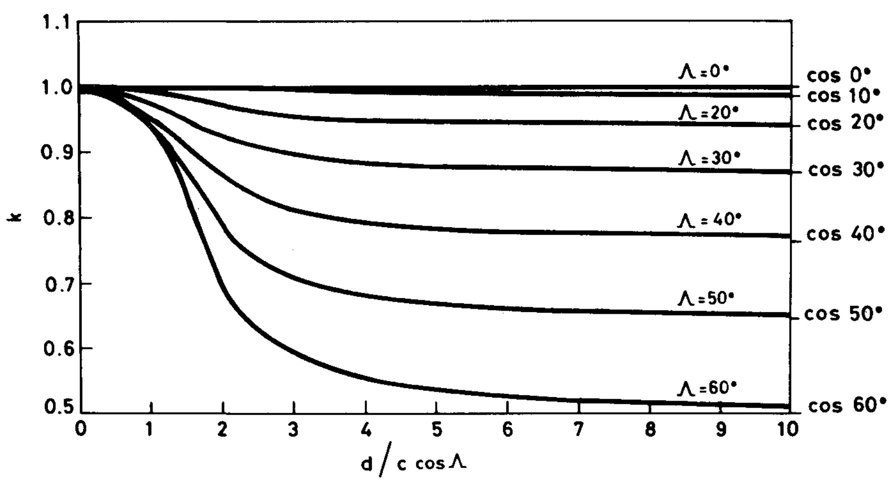

Fig. 3 Influence of the segment dimensions on the accuracy of the calculations.

troducing the effects of changes in vorticity distribution (compared with the case of infinite uniform wing) by changes in the effective angle of attack is well known and accepted. Such an approach is used, for example, in the classical lifting line models of finite wings.

The Discrete Model

In this section, a discrete model of an infinite swept wing is derived and solved. The discrete model also helps to prove that it is not the lift curve slope that is changed owing to the slope. The wing is modeled as a series of shifted rectangular segments, as shown by the broken lines of Fig. 2. The width of each segment is d, while its length is the chord c. The problem is solved by using the vortex lattice method. Therefore, the whole vortex sheet is represented by the thick "stairway shape" vortex line, as shown by Fig. 2. According to the vortex lattice method, the boundary condition is reduced to the requirement of nonpenetration of the resultant flow through the control point, which is positioned at the three-quarters chord point of the middle chord of the segment. The reason for choosing the three-quarters chord point as the point where the boundry condition should be satisfied is its exactness in the case of a two-dimensional airfoil with lift curve slope 2π. Therefore, in cases where the lift curve slope is different from 2π, this scheme should result in increasing errors in the calculations.

The lift per unit length in the y direction, L, can be expressed as follows:

$$L = L_0/k \qquad L_0 = \pi\rho U^2 c\alpha\cos\Lambda \tag{13}$$

The exact analytic expression for k (in the form of an infinite series) is given in Ref. 4. Figure 3 shows k as function of d/c and Λ. It is shown that as d/c approaches zero, k approaches unity, and the correct value of L_0 is obtained. This means that, although the wing is swept, the lift curve slope remains 2π. On the other hand, as d/c increases, an increase in L by a factor of $1/\cos\Lambda$ is asymptotically approached. In this case, an artificial correction factor of $\cos\Lambda$ in the lift curve slope is required.

Conclusion

As the result of the sweep of an infinite wing, the field of induced velocities over the wing is changed. This change is a result of a relative shift of the cross sections in the freestream direction and the appearance of vorticity components in the same direction. From the point of view of the two-dimensional behavior of the cross sections, it is better to describe the influence of sweep as a change in the effective angle of attack. This approach is equivalent to the classical method of the lifting line, when dealing with finite wings.

The correction in the effective angle of attack is more consistent and has better physical explanation than a correction in the lift curve slope of the profile. This is shown even more clearly in the case of a discrete model of the infinite swept wing.

References

[1] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity,* Addison-Wesley Publishing Co., Cambridge, Mass., 1955, pp. 250-251.

[2] Kuchemann, D., *The Aerodynamic Design of Aircraft,* 1st Ed., Pergamon Press, New York, 1978, pp. 111-113.

[3] McCormick, B. W., *Aerodynamics, Aeronautics and Flight Mechanics,* John Wiley and Sons, New York, 1979, pp. 281-283.

[4] Rand, O. and Rosen, A., "An Unsteady Prescribed Wake Model of the Aerodynamic Behavior of a Helicopter Rotor Having Curved Blades," Department of Aeronautical Engineering, Technion—Israel Institute of Technology, TAE Report 544 (in preparation).

Economical Influence Function Calibrations Using the Distributed Loads Code

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Introduction

THE Influence Function Method (IFM)[1] is a new technique for the prediction of store loads within an aircraft interference flowfield. A major step in the IFM procedure involves calculating the "influences" various segments of the store body have on the total forces and moments of the store. This process, known as "store calibration," requires that the total store forces and moments, as well as the local angle-of-attack distributions along the store length, be either calculated or measured at several axial positions as the store is traversed through a known "calibration" flowfield. The difficulty and expense involved in obtaining these force, moment, and local angle-of-attack distributions have been the major limitations of the IFM.

Typically, two approaches have been taken. In the first approach, a wind tunnel test is conducted in which a model of the store to be calibrated is traversed near a body that creates a known flowfield (supersonically, this may be done by traversing the store through a two-dimensional wedge shock wave; subsonically, determination of the known local flow angles is more difficult). This method requires wind tunnel support which is expensive. The second approach to store calibration involves theoretically calculating the total store forces and moments at several axial stations within a theoretically modeled flowfield (typically, calculations are required at 15-20 store axial positions in the flowfield). The PANAIR code[2,3] is the state-of-the-art computational technique usually used to obtain these force and moment predictions. However, PANAIR predictions at 15-20 locations are relatively expensive[4] and, therefore, are generally not very attractive (they would use approximately 75 min of CPU time using the version of the PANAIR Pilot code installed on the AEDC Amdahl 5860 computer).

Since neither dedicated calibration testing nor high-order computational aeropredictions are desirable from a cost viewpoint, an engineering methods prediction technique—the AEDC Interference Distributed Loads (IDL) code[5]—has been modified to predict store force and moment coefficients along an axial traverse through a simple calibration flowfield. IDL predictions were made for both the generic planar wing weapon (PWW) and the GBU-15 CWW stores at a total cost of 6 s of Amdahl CPU time. This represents a three-order-of-magnitude cost reduction over PANAIR calibrations. Preliminary comparisons of F-15 right inboard pylon flowfield predictions and grid loads on the PWW and GBU-15 stores in that flowfield indicate that the IDL-produced influence coefficients provide very accurate grid predictions which agree well with predictions derived from influence coefficients determined by calibration testing.

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